Image-Difference Measure Optimized Gamut Mapping

Jens Preiss and Philipp Urban;
Institute of Printing Science and Technology, Technische Universität Darmstadt;
Magdalenenstr. 2, 64289 Darmstadt, Germany

Abstract
Even though there is still room for improvement, recent perceptual image-difference measures show a prediction performance that makes them interesting to be used as objective functions for optimizing image processing algorithms. In this paper, we use a color enhanced modification of the Structural Similarity (SSIM) index for optimizing gamut mapping. An iterative algorithm is proposed that minimizes this measure for a given reference image subject to in-gamut images. Since distortions within remote image regions contribute independently to the measure a descent direction can be specified locally. The step-length is chosen to be a fraction of the just-noticeable-distance ensuring a decrease of the measure. Results show that the proposed approach preserves contrast and structural information of reference images. Some artifacts suggest modifications of the employed image-difference measure.

Introduction
Every color-reproduction workflow incorporates a color gamut mapping transformation to account for the limited ability of output devices to reproduce colors. A common objective of a gamut mapping transformation is to minimize the perceived difference between the original image and the reproduction.

To avoid artifacts, such as color banding, usually more than the non-reproducible colors have to be modified. In the early stage of gamut mapping research pixel-wise transformations have been investigated. A good overview of such gamut mapping methods is given by Morovic *et al.* [1]. In order to preserve local image contrasts, spatial gamut mapping has become of increasing interest in recent years [2, 3, 4, 5, 6]. An independent comparison of selected spatial gamut mapping methods can be found in [7].

Nearly all of these methods work within perceptual color spaces (e.g., hue linearized CIELAB or IPT color space [8]) but are based on heuristics (e.g., preserving hue is more important than preserving chroma). Calculating the gamut mapping operator by minimizing the perceptual difference to the original is rarely addressed in literature. Nakauchi et al. [9] proposed a method that minimizes a color image-difference measure which is very similar to the S-CIELAB-based image difference. Kimmel et al. [10] used a related measure but added gradients to the objective function in order to account for perceptual feature differences (e.g., color banding). Minimizing this objective function is similar to solving an Euler-Lagrange differential equation and finally (after a reformulation) a quadratic programming problem. Kimmel's approach is more of theoretical interest, since it requires devices with convex gamuts – a property that most real devices do not possess. Furthermore, Nakauchi and Kimmel et al. construct their objective function in a way that ΔL^*, Δa^* and Δb^* image-difference plains are treated separately and without considering the direction of difference. Optimizing such objective functions might lead, for instance, to adverse hue shifts.

In a recent publication Zolliker et al. [11] used a hue-enhanced modification of the SSIM index [12] to fuse images resulting from different gamut mapping algorithms. Visual experiments show that the resulting images are judged to be more similar to the originals than the results of the gamut mapping methods employed for the fusion process.

Further enhancements considering chromatic deviations were able to significantly improve the prediction performance of the SSIM index for gamut mapping distortions [13, 14]. Even though there is still much room for improvement, such image-difference measures could be directly used as objective functions for gamut mapping.

In this paper, we propose an algorithm that incorporates a slightly modified version of an image-difference measure described in [14] as an objective function for gamut mapping. Our aim is not only to present this method but also to learn more about the underlying image-difference measure.

The Image-Difference Measure
An image-difference measure maps two images and a set of parameters specifying the viewing conditions into a single number that is a prediction of the perceived image difference. The measure employed in this paper is called *color image-difference* (CID) measure. It is based upon an extension of the SSIM index [12] to color images [14]. For computing the CID measure between two images X,Y of the same size, they need to be normalized in a preceding step to reference viewing conditions by an image appearance model (e.g., to the viewing distance, luminance level, etc.) and then transformed into a working color space. We used the nearly perceptually uniform LAB2000HL opponent color space [15], which is moreover hue linear with respect to the Hung-Berns data [16], i.e. lines of constant perceived hue agree well with lines of predicted hue. The color space has a lightness axis "L", a red-green axis "a" and a blue-yellow axis "b" similar to the CIELAB color space that has some shortcomings with respect to hue linearity and perceptual uniformity.

The CID measure incorporates five terms to predict local lightness (l_L), chroma (l_C), and hue (l_H) differences as well as local lightness-contrast (c_L) and lightness-structure (s_L) differences. The terms are defined on rectangular windows covering the same regions within the images X and Y:

$$l_L(\mathbf{x},\mathbf{y}) = \frac{1}{c_1 \cdot \overline{\Delta L(\mathbf{x},\mathbf{y})}^2 + 1}, \quad (1)$$

$$c_L(\mathbf{x},\mathbf{y}) = \frac{2\sigma_{\mathbf{x}}\sigma_{\mathbf{y}} + c_2}{\sigma_{\mathbf{x}}^2 + \sigma_{\mathbf{y}}^2 + c_2}, \quad (2)$$

$$s_L(\mathbf{x},\mathbf{y}) = \frac{|\sigma_{\mathbf{xy}}| + c_3}{\sigma_{\mathbf{x}}\sigma_{\mathbf{y}} + c_3}, \quad (3)$$

$$l_C(\mathbf{x},\mathbf{y}) = \frac{1}{c_4 \cdot \overline{\Delta C(\mathbf{x},\mathbf{y})}^2 + 1}, \quad (4)$$

$$l_H(\mathbf{x},\mathbf{y}) = \frac{1}{c_5 \cdot \overline{\Delta H(\mathbf{x},\mathbf{y})}^2 + 1}, \quad (5)$$

where $\mathbf{x}$, $\mathbf{y}$ are the pixels within the windows, $\sigma_{\mathbf{x}}$ and $\sigma_{\mathbf{y}}$ are the corresponding Gaussian weighted standard deviations computed for the lightness component and $\sigma_{\mathbf{xy}}$ is the Gaussian weighted correlation of the lightness values between the windows. The parameters c_i are required to adjust the terms to the working color space and were set to $c_1 = c_4 = 0.002$, $c_2 = c_3 = 0.1$, and $c_5 = 0.008$ as proposed in [14]. $\overline{\Delta F(\mathbf{x},\mathbf{y})}$ denotes the Gaussian-weighted mean of the pixel-wise difference functions $\Delta F(x,y)$ computed for each pixel pair (x,y) within the window. These functions are defined as follows:

$$\Delta L(x,y) = L_x - L_y, \quad (6)$$

$$\Delta C(x,y) = \sqrt{a_x^2 + b_x^2} - \sqrt{a_y^2 + b_y^2}, \quad (7)$$

$$\Delta H(x,y) = \sqrt{(a_x - a_y)^2 + (b_x - b_y)^2 - \Delta C(x,y)^2}. \quad (8)$$

Applying the terms defined in eqs. (1)-(5) within sliding windows results in five difference maps $\mathbf{L}_L(X,Y)$, $\mathbf{C}_L(X,Y)$, $\mathbf{S}_L(X,Y)$, $\mathbf{L}_C(X,Y)$ and $\mathbf{L}_H(X,Y)$. They have the same size as the images to be compared (assuming an appropriate padding of the images' border pixels). The CID measure is then computed as follows

$$\mathbf{P}(X,Y) = 1 - \overline{\mathbf{L}_L(X,Y)\mathbf{C}_L(X,Y)\mathbf{S}_L(X,Y)\mathbf{L}_C(X,Y)\mathbf{L}_H(X,Y)}, \quad (9)$$

where the bar denotes the mean computed for the pixel-wise product of the maps. The CID measure is in the range of $\mathbf{P}(X,Y) \in [0,1]$. Larger values indicate larger image differences.

In contrast to the original SSIM index [12], chroma (l_C) and hue (l_H) differences are considered and the lightness difference term (l_L) is adjusted to a perceptually uniform color space. These modifications significantly improve the prediction performance of the SSIM index on a large visual dataset investigating gamut mapping distortions by paired comparison [14].

Methodology

We use the CID measure defined in eq. (9) as an objective function for gamut mapping, i.e. given the original image X we are looking for an image Z that solves the optimization problem

$$Z = \underset{Y \tilde{\subset} G}{\operatorname{argmin}}\ \mathbf{P}(X,Y), \quad (10)$$

where the expression $Y \tilde{\subset} G$ means that each pixel color of image Y is within the gamut $G \subset$ LAB2000HL of the output device.

We solve this constrained optimization problem by utilizing a special property of $\mathbf{P}$: Distortions within remote image regions contribute independently to the overall image difference.

To explain this in more detail, let (X_i,Y_i), $i \in I$ be a set of non-overlapping rectangular sub-images extracted from the image pair (X,Y) and I is a set of pixel locations indicating the sub-images' center pixels. X_i and Y_i cover the same region within the corresponding images. If the window size employed by $\mathbf{P}$ is

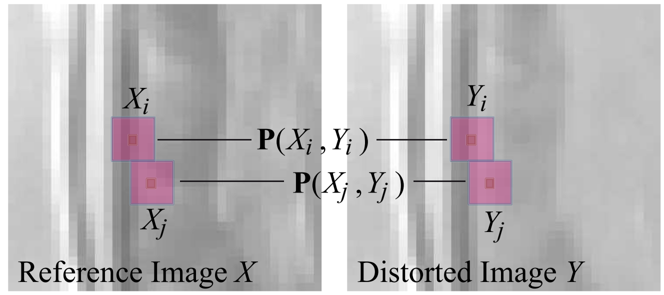

Figure 1. *Distortions resulting from changing the center pixels of sub-images (X_i,Y_i) and (X_j,Y_j) contribute independently to the overall image difference $\mathbf{P}(X,Y)$ if a 3×3 window is used to compute $\mathbf{P}$. The center of sub-images (X_i,Y_i) has a pixel position of $i = (i_1,i_2)$ and the center of sub-images (X_j,Y_j) has a pixel position of $j = (j_1,j_2) = (i_1+2, i_2+5)$.*

$n \times n$ pixels (n odd) then the size of the sub-images is $(2n-1) \times (2n-1)$. Figure 1 shows an example of such sub-images. The mentioned property can be written as follows

$$\begin{gathered}\mathbf{P}(X, Y + \sum_{i\in I} E^i(y_i)) = \\ \mathbf{P}(X,Y) + \frac{K}{N}\sum_{i\in I}\left[\mathbf{P}(X_i, Y_i + W(y_i)) - \mathbf{P}(X_i,Y_i)\right],\end{gathered} \quad (11)$$

where $E^i(y_i)$ is an image of the same size as Y. All pixel values of $E^i(y_i)$ are zero except at location i where the pixel value is $y_i \in \mathbb{R}^3$. $W(y_i)$ is an image of the same size as Y_i composed of zero-valued pixels except the center pixel with value y_i. N is the number of pixels in X or Y and $K = (2n-1)^2$.

In other words: By changing the center pixels of sub-images located in remote image regions the CID measure does not need to be recomputed completely. It is sufficient to compute the CID measure of the sub-images and upgrade the overall CID measure.

From this property the following implication can be derived

$$\mathbf{P}(X_i, Y_i + W(y_i)) \le \mathbf{P}(X_i,Y_i) \Rightarrow \mathbf{P}(X, Y + E^i(y_i)) \le \mathbf{P}(X,Y), \quad (12)$$

which allows us to construct a descent direction for the optimization by considering only small sub-images.

In this work, we use discrete optimization to solve the problem stated in eq. (10). To determine a suitable starting image for our iteration, the pixel colors of the reference image X must be transformed into the gamut G using a common gamut mapping algorithm (GMA), i.e. $Y = \text{GMA}(X) \tilde{\subset} G$. The in-gamut image Y is upgraded at pixel position i by solving the following discrete optimization problem for the sub-images X_i and Y_i

$$Y(i) = Y(i) + \underset{z \in H(i)}{\operatorname{argmin}}\ \mathbf{P}(X_i, Y_i + W(z)), \quad (13)$$

$$H(i) = \left\{ z \in \{-\alpha, 0, \alpha\}^3 \mid Y(i) + z \in G \right\},$$

where α is a fraction of the just noticeable distance (JND) and $Y(i)$ is the LAB2000HL color of the i-th pixel of Y. The value α can be chosen independently of the pixel color $Y(i)$ in a perceptually uniform color space. Since $H(i)$ has at most $3^3 = 27$ elements a brute force approach is reasonable. Without going into detail, it is worth mentioning that the five difference maps required to determine $\mathbf{P}(X_i, Y_i)$ need to be modified only slightly for computing $\mathbf{P}(X_i, Y_i + W(z))$, $z \in H(i)$. The numerical effort of upgrading these difference maps is rather small.

Solving problem (13) for all pixels of Y is denoted as one iteration. Algorithm 1 shows all steps as a pseudo-code keeping the previous terminology.

Algorithm 1 – CID-OPTIMIZED GAMUT MAPPING

INPUT: Gamut G, reference image X (m_1-rows, m_2-columns)

$Y = \mathrm{GMA}(X) \tilde{\subset} G$, where GMA is a gamut mapping algorithm

REPEAT

  $Y' = Y$

  FOR EACH $i \in \{1, \ldots, m_1\} \times \{1, \ldots, m_2\}$

    $Y(i) = Y(i) + \mathrm{argmin}_{z \in H(i)} \mathbf{P}(X_i, Y_i + W(z))$

  END FOR

UNTIL $\mathbf{P}(X, Y') - \mathbf{P}(X, Y) < \varepsilon$

OUTPUT: Y

Please note that the `FOR`-loop can be parallelized because of property (11). The algorithm is terminated if the image-difference improvement between two iterations falls below $\varepsilon > 0$.

Results and Discussion

The primary focus of this paper is not to develop a new gamut mapping algorithm that improves the state of the art. Our aim is rather to propose and investigate a method to incorporate even complex image-difference measures for gamut mapping by utilizing a simple property. It should be noted that the performance of this approach is limited by the prediction performance of the employed measure. Lissner et al. [14] showed that the CID measure has still much room for improvement and it is beyond the scope of this work to enhance the accuracy of the image-difference measure for obtaining better gamut mapping results. It is worth mentioning that we can replace the CID measure employed in this work by any measure that possesses property (11).

The major aim of the experiments is to investigate the convergence behavior with respect to the number of required iterations and local minima. We are also interested if the starting images resulting from existing GMAs can be improved by the optimization and if there are any artifacts in the resulting images.

Experiments

We mapped five images shown in figures 4 and 5 onto a small newspaper gamut by minimizing the CID measure defined in eq. (9). The gamut is specified by the `USNewsprintSNAP2007.icc` profile and is shown in figure 2. We used a very small gamut possessing a black point with high lightness to ensure large visible distortions. Two point-wise GMAs were used to map the reference images into the gamut to obtain a valid starting image for the optimization:

1. **CLIPSLIN**: Chroma and lightness clipping towards the middle-gray value.
2. **SGCK**: Chroma-dependent sigmodial lightness mapping followed by knee scaling towards the CUSP [1].

Both GMAs were applied within the LAB2000HL color space [15]. Mappings were performed along lines of constant hue.

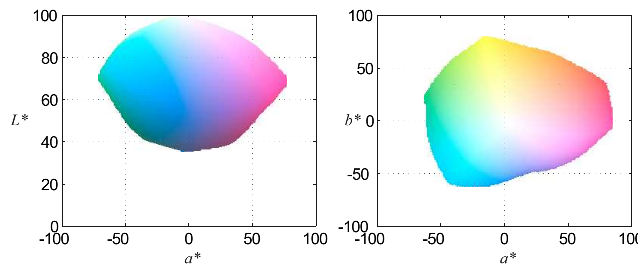

Figure 2. *Media-relative CIELAB plot of the newspaper gamut used in the experiments.*

Normalizing the input images to reference viewing conditions results in an improved prediction performance of the CID measure [14]. In this paper, the normalization step was omitted and all calculations were performed on the raw images. An 11×11 window was used. For the optimization we employed a step length (i.e. α – see eq. (13)) that corresponds to approx. 0.4 CIEDE2000 units – a color difference that can be assumed to be below JND on typical displays.

Discussion

Our results show that the proposed optimization method converges to solutions that depend on the starting image, i.e., on the GMA employed to map the reference image into the gamut. This is also reflected by the CID values resulting from the optimization as can be seen from figure 3. This figure shows also that after approximately ten iterations the predicted image difference remains nearly constant. In average, twelve (and at most sixteen iterations) were necessary to terminate the optimization for $\varepsilon = 0.001$ (see algorithm 1).

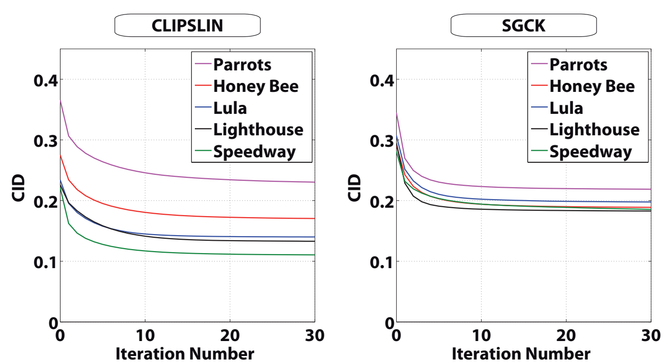

Figure 3. *Image difference vs. iteration number*

The optimized images show much smaller predicted image differences to the originals than the starting images. However, does the optimization minimize also the perceived image difference? This question is clearly related to the prediction performance of the CID measure. The answer tells us whether the optimization approach is reasonable for today's state of the art image-difference measures. We conducted a visual experiment employing seventeen unbiased color-normal observers (six females and eleven males) to investigate the visual effects of the optimization for our test images and GMAs. The starting image (obtained by CLIPSLIN or SGCK) and the corresponding optimized

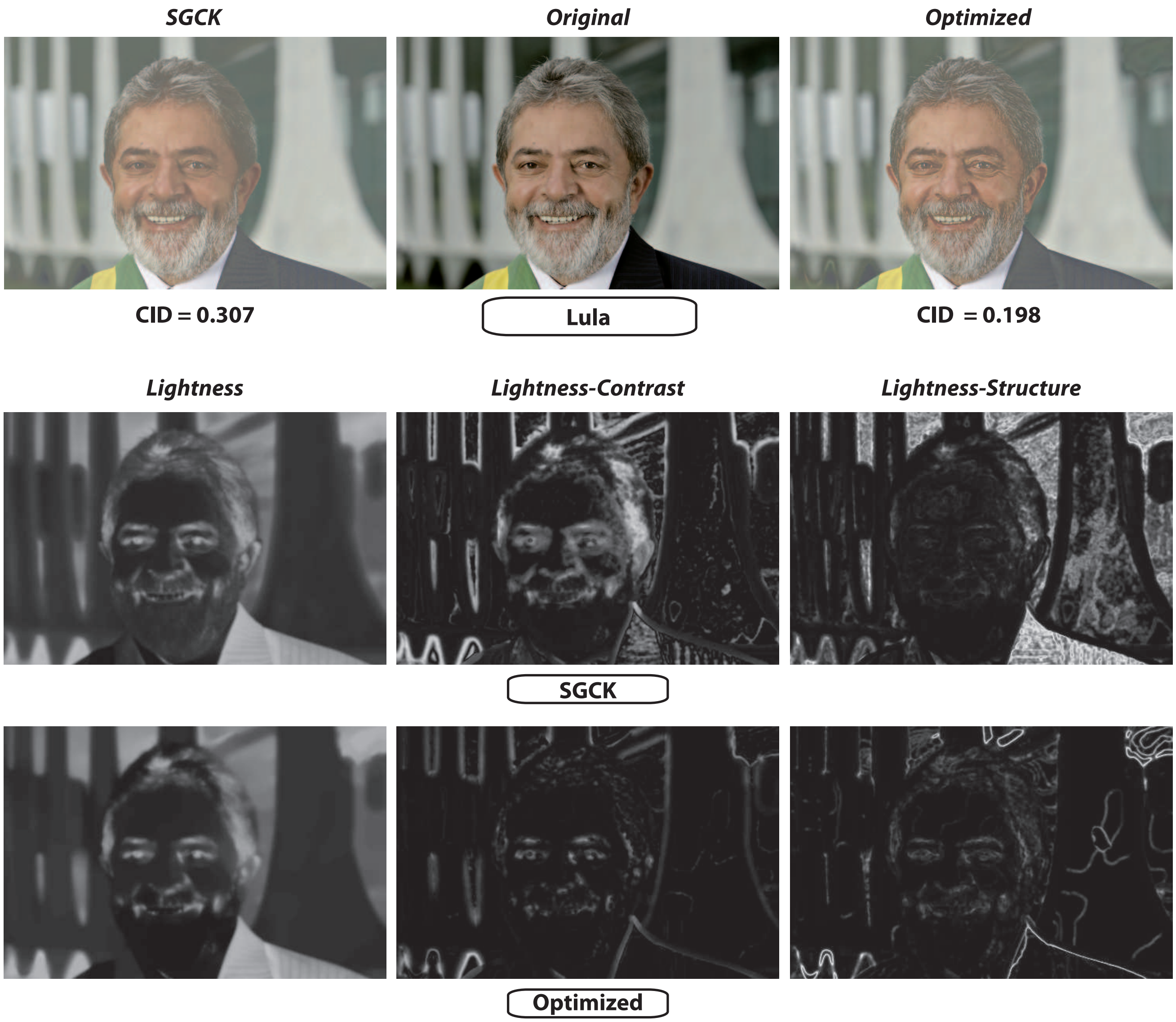

Figure 4. The top row shows the original Lula image (middle) gamut mapped by SGCK (left) and the corresponding optimized image (right). The lightness, lightness-contrast, and lightness-structure difference maps indicate how these image-difference attributes are changed by the optimization. The color-difference maps are negligible.

image were shown together with the reference image on a calibrated display. Subjects were asked to judge what image is more similar to the reference. Each image triplet was shown twice to each observer. In 82.6% of all cases the optimized image was selected. 87.1% of the decisions preferred the optimized images if the starting image was computed by CLIPSLIN. 78.2% favor the optimized images if the starting image was computed by SGCK. These results are significant ($p < 0.01$). The perceived image difference was clearly reduced by the optimization. If state of the art spatial-gamut mapping algorithms might also be further enhanced by minimizing the CID measure is an open question and shall be left to future comparison experiments.

Finally, we analyzed the resulting images with respect to artifacts. Lightness artifacts were indeed found, particularly in dark areas. Their occurrence highly depend on the starting image, i.e., the GMA used to generate the initial in-gamut image. Optimizing the *Lula* image results in some artifacts that can be seen in figure 4. One interesting artifact is shown at the jacket's pinstripes that are lightness-inverted to the background compared to the reference image. The reason is the starting image that already shows this artifact. Even though the CID-optimization clearly improved contrast and structure (which is well illustrated by the corresponding difference maps above) this type of artifact affects only the lightness difference map. The lightness-inversion is neither a contrast nor a structural deviation. The optimization found a local minimum in this area.

Another artifact is found in the upper right and lower left corner of the optimized *Lula* image. These artifacts are caused by another local minimum and can be clearly detected by the structure difference map. Furthermore, we found some color ringing e.g., in the *Speedway* image above the helmets. We believe that the optimization has too many degrees of freedom in the color channels. Adding a structural term for chroma or hue to the employed CID measure would probably solve this problem.

Conclusions

In this paper, gamut mapping is considered as a constrained optimization problem. An image-difference measure is minimized for a given reference image subject to all in-gamut images. We employed a color enhanced modification of the structural similarity (SSIM) index, called color image-difference (CID) measrue, as the objective function. Utilizing that distortions within remote image regions contribute independently to the measure, an iterative algorithm to solve the constrained optimization problem is proposed. Starting from an in-gamut image computed by an existing gamut mapping algorithm (GMA), in each iteration step every pixel is changed by solving a constrained discrete optimization problem locally. Results show distinct improvements with respect to retaining contrast and structural information. A visual experiment validated that the optimized images have significantly smaller perceived image differences to the originals than the starting images. Spatially-confined artifacts resulting from local minima uncover new research directions to improve the underlying image-difference measure.

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Author Biography

Jens Preiss received his diploma in Physics (equivalent to a M.S.) from the University of Freiburg (Germany) in 2010. He is currently a research assistant at the Institute of Printing Science and Technology, Technische Universität Darmstadt (Germany), where he works as a doctoral candidate in the area of color and imaging science.

Philipp Urban has been head of an Emmy-Noether research group at the Technische Universität Darmstadt (Germany) since 2009. His research focuses on color science and spectral imaging. From 2006–2008 he was a visiting scientist at the RIT Munsell Color Science Laboratory. He holds a MS in mathematics from the University of Hamburg and a PhD from the Hamburg University of Technology (Germany).

Figure 5. *The optimized images (right column) used the CLIPSLIN images (left column) as starting images for the iteration. The middle column shows the reference image.*