Gamut Boundary Determination by Sampling an Inverse Printer Model

Philipp Urban, Rolf-Rainer Grigat
Ratio Entwicklungen GmbH, Technical University Hamburg-Harburg
Hamburg, Germany

Abstract
In this paper we present a novel method to calculate the gamut boundary of a color printing device using a sparse number of measured values. This method belongs to the SMGBD (Segment Maxima Gamut Boundary Descriptor) family. At first, we construct an inverse printer model based on a set of printed color patches. This model allows us to find the colorant values that have to be sent to the printer in order to obtain a given color appearance, or to show that such a colorant value does not exist. Then we construct a GBD (Gamut Boundary Descriptor) with the measured color patches, using the segment maxima method. After calculating the GBD every segment contains exactly one color value, which might not necessarily be part of the device gamut boundary. Therefore we sample along the line determined by the contained color value and the middle grey value \((L, a, b) = (50, 0, 0)\) for every segment, using a bisection method based on the inverse printer model, and store the printable color value on the line with the largest distance from the middle grey value in the GBD. Due to this approach practical gamut boundaries can be calculated based on a sparse number of measured colors. Measurement results and a comparison with other methods are given in the text.

Introduction
The reproduction of colors by a printer is a two stage process. The first stage consists of characterizing the device, in order to determine the colorant values which obtain a desired color appearance (inverse printer model). In the second stage colors which are physically not reproducible by the printer must be transformed to printable colors in a faithful way (gamut mapping). The mapping of not printable colors into the amount of physically reproducible colors requires the knowledge of the gamuts boundary. Additionally, it is important to store the gamut boundary information in a compact form to allow an easy and fast access for gamut mapping methods.

In recent years different methods have been developed to calculate the gamut surface. Some of these methods do not take the mapping properties of the appropriate printer model into account. Even if the set of spectrophotometrically measured colorant values is located also inside the hyper-cube, only the printer model which will be used for characterization determines the shape of the gamut (natural gamut boundaries). The presented method exploits the properties of the printer model mapping and provides a GBD which allows easy access to shape information.

This paper is organized as follows: At first, we will give an overview about printer characterization and define the indicator function \(I_{\tilde{F}} : \text{CIELab} \rightarrow \{-1, 1\}\) based on the inverse printer model \(\tilde{F}\). \(I_{\tilde{F}}(\theta)\) returns 1 if there exist colorant values that reproduce \(\theta\), otherwise \(-1\). In the next section the SMGBD method of Morovic [8] will be sketched. Then we will describe the Segment Maxima Sampling GBD (SMSSGBD) method. This bisecting algorithm enhances the SMGBD method by sampling an inverse printer model using the above indicator function \(I_{\tilde{F}}\). Finally some measurement results and conclusions will be given.
Inverse Printer Model

A colorimetric printer model \( F_P : \Omega_{ink} \rightarrow \text{CIELab} \) is a mapping which approximately describes the behavior of a printer for a given paper type (\( \Omega_{ink} \) specifies the colorant values of the printer, e.g. RGB, CMY, CMYK, CcMmYK). For every colorant value \( \chi \in \Omega_{ink} \) which will be sent to the printer \( F_P \) returns the appropriate color appearance \( \vartheta \in \text{CIELab} \), which will be generated by the printer. In general \( F_P(\Omega_{ink}) \neq \text{CIELab} \) applies. We denote the printer media gamut by \( F_P(\Omega_{ink}) \subset \text{CIELab} \), depending on the appropriate printer model. The better the model describes the printer the better \( F_P(\Omega_{ink}) \) describes the printer media gamut.

The inverse question is even more interesting: What colorant value has to be sent to the printer in order to obtain a given color appearance?

As in general there is more than one colorant value which maps to the desired color appearance, the printer model is not invertible (\( F_P \) is not injective). This applies especially for printers using more than 3 ink types. To solve this problem special criterions will be established to select one specific colorant value out of the set of possible colorant values (e.g. black generation). The inversion of a printer model is in general not unique. Therefore there exists a set of “inverse” printer models for every printer model \( F_P \).

\[
\mathbb{F}_{F_P} := \left\{ \tilde{F} : F_P(\Omega_{ink}) \rightarrow \Omega_{ink} \right\} \\
\forall \vartheta \in F_P(\Omega_{ink}) : F_P(\tilde{F}(\vartheta)) = \vartheta .
\] (1)

Because for color appearances \( \vartheta \in \text{CIELab} \setminus F_P(\Omega_{ink}) \) for every inverse printer model \( \tilde{F} \in \mathbb{F}_{F_P} \) there is an indicator function \( I_{\tilde{F}} : \text{CIELab} \rightarrow \{-1, 1\} \)

\[
I_{\tilde{F}} : \left\{ \text{CIELab} \rightarrow \{-1, 1\} \right\} \\
\vartheta \mapsto \begin{cases} 1, & \exists \chi \in \Omega_{ink} : \chi = \tilde{F}(\vartheta) \\ -1, & \text{else} . \end{cases}
\] (2)

For a given printer model \( F_P \) all indicator functions are equal \( I := I_{\tilde{F}} = I_{\tilde{F}'} \), with \( \tilde{F}_1 \neq \tilde{F}_2 \).

In recent years different models (e.g. Neugebauer [9], Yule-Nielson [11] and their derivatives, Urban-Grigat [10]) have been developed to characterizing printers. The printer models will be constructed using a set of printed and measured color patches with known colorant values \( \Omega_{ink}^{ext} \subset \Omega_{ink} \) (e.g. IT8.7/3 for \( \Omega_{ink} = \text{CMYK} \)).

Techniques to invert this models can also be found in the literature (Cholewo [3], Mahy-Delabastita [5], Urban-Grigat [10]).

Segment Maxima GBD Method (SMGBD)

This method was proposed by Morovic [8] and enables the determination of a 3D printer media gamut in the CIELab color space by calculating a GBD as follows:

1. The CIELab color space will be transformed into spherical coordinates with center in the middle gray value \( (L_M, a_M, b_M) = (50, 0, 0) \), using following formulae

\[
r = \sqrt{(L - L_M)^2 + (a - a_M)^2 + (b - b_M)^2} \\
\phi = \arctan \left( \frac{b - b_M}{a - a_M} \right) \\
\theta = \arctan \left( \frac{L - L_M}{\sqrt{(a - a_M)^2 + (b - b_M)^2}} \right)
\] (3) (4) (5)

\( \phi \in [-\pi, \pi] \) is the hue angle and \( \theta \in [-\frac{\pi}{2}, \frac{\pi}{2}] \) is the angle in a plane of constant \( \phi \).

2. The CIELab color space will be divided into \( n \times n \) segments

\[
S_{ij} := \left\{ \vartheta \left| \vartheta_\phi \in \left[ -\pi + \frac{2(i-1)\pi}{n}, -\pi + \frac{2i\pi}{n} \right] \right. \right. \\
\bigg\} \\
\vartheta_\phi \in \left[ -\frac{\pi}{2} + \frac{(j-1)\pi}{n}, -\frac{\pi}{2} + \frac{j\pi}{n} \right], \quad i, j = 1, \ldots, n
\] (Figure 1c). The segment \( S_{ij} \) corresponds to the entry \( A_{ij}^{GBD} \) of the GBD matrix \( A^{GBD} \) (Figure 1b).
3. Each of the measured reference colors \( \Omega_{\text{ref}} \in \Omega_{\text{ink}} \) will be transformed into spherical coordinates using equations 3 - 5. For every segment \( S_{ij} \) the point \( \vartheta \in S_{ij} \) with the largest \( r \)-value \( \vartheta_{r} = \max_{\vartheta \in S_{ij}} \vartheta_{r} \) will be stored in the \( A_{ij}^{GBD} \) entry of the GBD matrix \( A^{GBD} \).

4. GBD matrix entries for segments without colors will be interpolated from nearest GBD matrix entries.

\[
B_{ij}(\lambda) = M + \lambda \frac{C_{ij} - M}{\|C_{ij} - M\|_2} = M + \lambda \frac{C_{ij} - M}{(A_{ij}^{GBD})_r}
\]  

(7)

And the following sequence \((u_m, v_m)\) will define the bisecting iteration for segment \( S_{ij} \) using the indicator function \( \mathcal{I} \) from equation 2:

\[
(u_0, v_0) := \begin{cases} 
((A_{ij}^{GBD})_r, 0) & \mathcal{I}(B_{ij}(A_{ij}^{GBD})_r) = -1 \\
((A_{ij}^{GBD})_r, r_{\text{max}}) & \mathcal{I}(B_{ij}(A_{ij}^{GBD})_r) = 1 
\end{cases}
\]

For \( m = 1, 2, \ldots \) and \( x := \frac{u_{m-1} + v_{m-1}}{2} \):

\[
(u_m, v_m) := \begin{cases} 
(x, v_{m-1}) & \mathcal{I}(B_{ij}(x)) \cdot \mathcal{I}(B_{ij}(v_{m-1})) = -1 \\
(u_{m-1}, x) & \text{else}
\end{cases}
\]

The iteration stops, if the distance between \( u_m \) and \( v_m \) falls below a threshold \( \epsilon : |u_m - v_m| < \epsilon \) (\( \epsilon = 1 \) is mostly sufficient).

After termination the new sampling maximum point

\[
C_{ij}^{\text{new}} := M + u_m \frac{C_{ij} - M}{\|C_{ij} - M\|_2}
\]

will be stored in the GBD matrix in spherical coordinates using equations 3 - 5.

Using the same GBD structure as the SMGBD algorithm the same methods can be also used to access the surface information, like CLGB (Constrained Line Gamut Boundary Method) or FSLGB (Flexible Sequential Line Gamut Boundary Method) developed by Morovic [8].

**Results**

We have reviewed the algorithm on a HP Deskjet 990Cxi RGB-printer printing at first 17376 test color patches on normal paper. We have selected the set of color patches as follows:

\[
\Omega_{\text{nall}} := \{0, 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48, 52, 56, 60, 64, 68, 72, 76, 80, 84, 88, 92, 96, 100\}^3 \subset RGB.
\]

To calculate the inverse printer model we have chosen the "Printer Characterization Using Adaptive Constraint Optimization" method from Urban-Grigat [10]. Measurements of color patches have been made with an X-Rite DTP41 spectrophotometer, which used D50 illuminant, a 0°/45° measuring geometry and the 2° standard observer. The following 4 subsets of \( \Omega_{\text{nall}} \) have been chosen for gamut boundary calculation:

\[
\Omega_{\text{leaf}}^{\text{leaf}} := \{0, 8, 16, 24, 32, 40, 48, 56, 64, 72, 80, 88, 96, 100\}^3 \subset \Omega_{\text{nall}}
\]
\[
\Omega_{\text{leaf}}^{\text{gray}} := \{0, 12, 24, 36, 48, 60, 72, 84, 100\}^3 \subset \Omega_{\text{nall}}
\]
\[
\Omega_{\text{leaf}}^{\text{gray2}} := \{0, 16, 32, 48, 64, 80, 96, 100\}^3 \subset \Omega_{\text{nall}}
\]
\[
\Omega_{\text{leaf}}^{\text{gray3}} := \{0, 20, 40, 60, 80, 100\}^3 \subset \Omega_{\text{nall}}.
\]
We have compared the SMSGBD method to the SMGBD method using $32 \times 32$ segments. At first, two GBDs for $\Omega_{\text{test}}$ have been calculated using both methods. These GBDs serve as the reference gamut boundary information for the appropriate method. After calculating the gamut boundary for a set of test colorants using one method the results have been compared with the reference GBD of the same method. For each segment center the gamut boundary point has been determined for both GBDs. The $\Delta E_{94}$ distance between these two points has been calculated. For every set of colorants and for each method this result in $32 \times 32$ distance values. Figure 3 shows the mean error and figure 4 the maximal error.

![Mean Error](image)

**Figure 3:** Left bars SMSGBD method, right bars SMGBD method

![Max Error](image)

**Figure 4:** Left bars SMSGBD method, right bars SMGBD method

**Conclusion**

We have presented a new method (SMSGBD) to determine gamut boundaries by sampling an inverse printer model. This approach takes the properties of the printer model into account and stores the surface information in an GBD which is easy to address by gamut mapping algorithms. The quality of the SMSGBD method's results can be improved by using a more accurate printer model. Increasing the number of segments allows a more precise representation of the gamut boundary.

**References**


**Biography**

Philipp Urban was born in 1975 in Wroclaw, Poland. He received the dipl. math at the University of Hamburg, Germany in 1999. Since 1999 he is part of the research group “Vision Systems” of the Technical University Hamburg-Harburg, Germany, and works for “Ratio Entwicklungen GmbH”, ICC-member. His research interests are color management, gamut boundary calculation, device calibration and characterization.