

How Perceptually Uniform Can a Hue Linear Color Space Be?

Ingmar Lissner and Philipp Urban;
Institute of Printing Science and Technology, Technische Universität Darmstadt;
Magdalenenstr. 2, 64289 Darmstadt, Germany

Abstract

We propose a numerical method to determine a transformation of a color space into a hue linear color space with a maximum degree of perceptual uniformity. In a first step, a transformation of the initial color space into a nearly perceptually uniform space is computed using multigrid optimization. In a second step, a hue correction is applied to the resulting color space while preserving the perceptual uniformity as far as possible. The two-stage transformation can be stored as a single lookup table for convenient usage in gamut mapping applications.

We evaluated our approach on the CIELAB color space using the CIEDE2000 color-difference formula as a measure of perceptual uniformity and the Hung and Berns data as a reference of constant perceived hue. Our experiments show a mean disagreement of 5.0% and a STRESS index of 9.43 between CIEDE2000 color differences and Euclidean distances in the resulting hue linear color space. Comparisons with the hue linear IPT color space illustrate the performance of our method.

Introduction

Psychophysical experiments show that observers favor hue-preserving gamut mapping algorithms. Maintaining the perceived hue is therefore an important objective in gamut mapping [1]. Hue linear color spaces, in which the lines of constant hue are straight lines, allow simple access to constant hue curves. Another desirable property for gamut mapping is perceptual uniformity of the color space, meaning that Euclidean distances agree with perceived distances. This is important for adjusting the degree of compression or for preserving contrast ratios. A gamut representation in the perceptually non-uniform CIELAB color space may lead to contrast ratio changes if highly chromatic gamut regions and regions close to the gray axis are treated similarly.

In addition, CIELAB is not hue linear, which is especially evident in the blue region (see Fig. 1) [2]. If a gamut mapping is performed in CIELAB, a hue correction of this region is strongly recommended [3, 4]. Other color spaces are especially designed to be hue linear, such as the IPT color space [5], but they exhibit a lack of perceptual uniformity. Color order systems, such as the Munsell system, are also designed to be hue linear, but they cover rather low chroma regions.

Unfortunately, there are many indicators that a perceptually uniform color space does not exist [6, 7, 8, 9]. To find a space with optimal perceptual uniformity, Urban et al. proposed a method to transform non-Euclidean into Euclidean color spaces with minimal isometric disagreement [10, 11]. The resulting color spaces show a high degree of perceptual uniformity, provided that the underlying color-difference formulas accurately reflect perceived color differences.

Constant hue curves [3, 4] plotted in these approximately per-

ceptually uniform color spaces reveal a significant lack of hue linearity (as shown for the LAB2000 space in Fig. 1). As a consequence, these color spaces are not recommended for gamut mapping — unless colors are mapped along curved trajectories, which requires much greater computational effort.

As already mentioned, a hue linear color space with a maximum degree of perceptual uniformity would be beneficial for gamut mapping applications. This requires the creation of a new color space that combines the *local* property of perceptual uniformity with the *global* property of hue linearity. Instead of fitting the parameters of analytical functions to visual data, a numerical transformation based on lookup tables is used in this paper.

To illustrate the basic concept of our method, we create a transformation of the CIELAB color space using the CIEDE2000 [12] color-difference formula as a measure of perceptual uniformity and the Hung and Berns data [3, 4] as a reference of constant perceived hue. Other color spaces such as the CIECAM02 [13] space and other color-difference formulas such as CIE94 [14], CMC [15] or improved versions of these formulas [16, 17] can be used equivalently.

The Color Space Transformation

Our initial color space is perceptually non-uniform and not hue linear. We assume that a color-difference formula is defined on this space, and that its color-difference estimations accurately reflect perceived color differences.

The proposed method is a two-stage transformation of the initial color space. The first transformation maps the color space to a Euclidean space (Euclidean metric) with minimal isometric (length-preserving) disagreement with respect to the color-difference formula. The second transformation maps the resulting color space to a hue linear space while keeping the disagreement small. These transformations can be combined into a single color lookup table for usage in gamut mapping algorithms.

In this paper, we use CIELAB as our initial color space, because it is well known and used in many industrial standards. The CIEDE2000 color-difference formula is used to estimate perceived color differences in CIELAB. The transformations can be summarized as follows:

$$\begin{array}{lcl} \mathcal{T}_{00} : & \text{CIELAB} & \xrightarrow{\text{Stage 1}} \text{LAB2000} \\ \mathcal{T}_{00,\text{HL}} : & \text{LAB2000} & \xrightarrow{\text{Stage 2}} \text{LAB2000}_{\text{HL}}, \end{array} \quad (1)$$

where LAB2000 [10] and LAB2000_{HL} are Euclidean color spaces with minimal isometric disagreement with respect to CIEDE2000, and LAB2000_{HL} is hue linear. The transformations can be turned into a single transformation by composition: $\mathcal{T} = \mathcal{T}_{00,\text{HL}} \circ \mathcal{T}_{00}$.

Stage 1: Perceptual Uniformity

The transformation of the CIELAB color space into a Euclidean space with respect to the CIEDE2000 color-difference formula has been described by Urban et al. [10]. We will therefore only sketch the method roughly. The color space transformation for CIELAB and CIEDE2000 is available online [18].

Because CIEDE2000 treats lightness differences independently of hue and chroma differences, the a^*b^* -plane is treated separately from L^* . The L^* coordinate is transformed into the perceptually uniform lightness coordinate L_{00}^* by numerically integrating the CIEDE2000 formula along the lightness axis. The result is a one-dimensional lookup table.

The a^*b^* -plane is transformed using a two-dimensional lookup table. This table is calculated using multigrid optimization, starting from two regular grids whose vertices cover the a^*b^* -plane. These grids are designed such that each mesh of a grid encloses exactly one vertex of the other grid. The distance between any two neighboring vertices does not exceed five CIELAB units, the threshold below which CIEDE2000 correlates well with perceived differences [19]. For each mesh, the CIEDE2000 differences are calculated between its four vertices and the enclosed vertex of the other grid. The resulting four color differences are stored and remain unchanged during the subsequent multigrid optimization.

In every iteration of the optimization, the vertices of a grid are shifted based on the meshes of the other grid. The objective is to decrease the disagreement between the stored CIEDE2000 differences and the corresponding Euclidean distances. In the first iteration, the vertices of the first grid are shifted based on the meshes of the second grid. In the second iteration, the vertices of the second grid are shifted based on the meshes of the first grid. The optimization continues with alternating grids until the change between subsequent iterations is sufficiently small.

The two-dimensional lookup table is then created by mapping the vertices of either starting grid to the vertices of the corresponding optimized grid. Intermediate points are computed using bilinear interpolation. Figure 1 shows a starting grid in CIELAB and the corresponding grid in the LAB2000 space resulting from the multigrid optimization (grids in gray).

The resulting color space transformation \mathcal{T}_{00} consists of a one-dimensional (lightness) and a two-dimensional lookup table:

$$\mathcal{T}_{00} : \begin{cases} \text{CIELAB} & \mapsto \text{LAB2000} \\ (L^*, a^*, b^*) & \mapsto (L_{00}^*(L^*), a_{00}^*(a^*, b^*), b_{00}^*(a^*, b^*)). \end{cases}$$

Our lookup tables consist of 10^5 grid points (L^*) and 513×513 grid points (a^*b^*). Although the transformation could be expressed as a single three-dimensional lookup table, this would greatly increase its memory requirements without adding to the accuracy. Further details including the mathematical background and a performance evaluation of the transformation are provided by Urban et al. [10].

Stage 2: Hue Linearity

The second step of our transformation maps the curves of constant hue to straight lines while preserving the perceptual uniformity as far as possible. L_{00}^* remains our lightness coordinate — the lightness values are not changed.

The transformation uses interpolated and extrapolated curves of constant perceived hue based on the Hung and Berns data [3]. The data are available online [20]. They comprise 360 curves with

equidistant starting angles $\phi^0 = \{0, 1, 2, \dots, 359\}$ and only correct the hue in the blue CIELAB region. The data can be replaced by more accurate data if available.

Each curve of constant hue consists of 151 points with chroma values $\{0, 1, \dots, 150\}$. The data are stored as a matrix

$$\mathbf{H} = \{h_{i,j}\}_{151 \times 360}, \quad (2)$$

whose columns represent curves of constant hue with starting hue angles $\phi^0 = h_{0,j}$. The data can be used as a lookup table to compute a hue-corrected version of the CIELAB color space by mapping $h_{i,j} \mapsto h_{0,j}$. Intermediate points are computed using bilinear interpolation.

A hue-corrected version of the LAB2000 color space is determined accordingly with a lookup table transformation mapping curves of constant hue to straight lines. This hue correction is not unique — two degrees of freedom can be utilized to find a transformation that optimally preserves Euclidean distances in LAB2000: 1. we can allow the hue correction to change chroma, 2. we can vary the hue angles of the straight lines used in the mapping (see Fig. 2 (a)).

In this paper we focus on optimizing the hue angles, because the chroma values of points on constant hue curves are already compressed by \mathcal{T}_{00} . The hue angle optimization is a constrained optimization problem, which will be described in the following.

LAB2000 Transformation

First, we transform the curves of constant hue in \mathbf{H} to their a^*b^* -coordinates and map the resulting colors to the LAB2000 color space using \mathcal{T}_{00} as described in Stage 1. Note that any L^* can be used, because the constant hue data we consider are independent of the lightness level. Figure 1 shows the curves of constant hue (red lines) in CIELAB and the transformed curves in LAB2000. The resulting curves can be expressed in polar coordinates as:

$$\begin{aligned} \mathbf{C}_{00} &= \{c_{i,j}^{00}\}_{151 \times 360}, \\ \mathbf{H}_{00} &= \{h_{i,j}^{00}\}_{151 \times 360}, \end{aligned} \quad (3)$$

where the entries of \mathbf{H}_{00} are hue angles and the entries of \mathbf{C}_{00} are the corresponding chroma values in LAB2000. The vectors $(c_{0,j}^{00}, h_{0,j}^{00}), \dots, (c_{150,j}^{00}, h_{150,j}^{00})$ represent lines of constant perceived hue and correspond to the column vector $(h_{0,j}, \dots, h_{150,j})$ of \mathbf{H} in CIELAB.

Hue Angle Optimization

A mapping of the resulting constant hue curves to straight lines with hue angles $\phi = \{\phi_0, \dots, \phi_{359}\}$ can be performed by a two-dimensional color lookup table. The chroma remains unchanged: $c_{i,j}^{00} \mapsto c_{i,j}^{00}$, and the hue is corrected: $h_{i,j}^{00} \mapsto \phi_j$. Intermediate colors are transformed using bilinear interpolation.

Our goal is to find hue angles $\phi^{\text{opt}} = \{\phi_0^{\text{opt}}, \dots, \phi_{359}^{\text{opt}}\}$ that minimize the mean disagreement $d(\phi)$ between CIEDE2000 color differences in CIELAB and Euclidean distances in the new space. This task can be expressed as a constrained optimization problem:

$$\phi^{\text{opt}} = \underset{\phi}{\operatorname{argmin}} d(\phi), \quad (4)$$

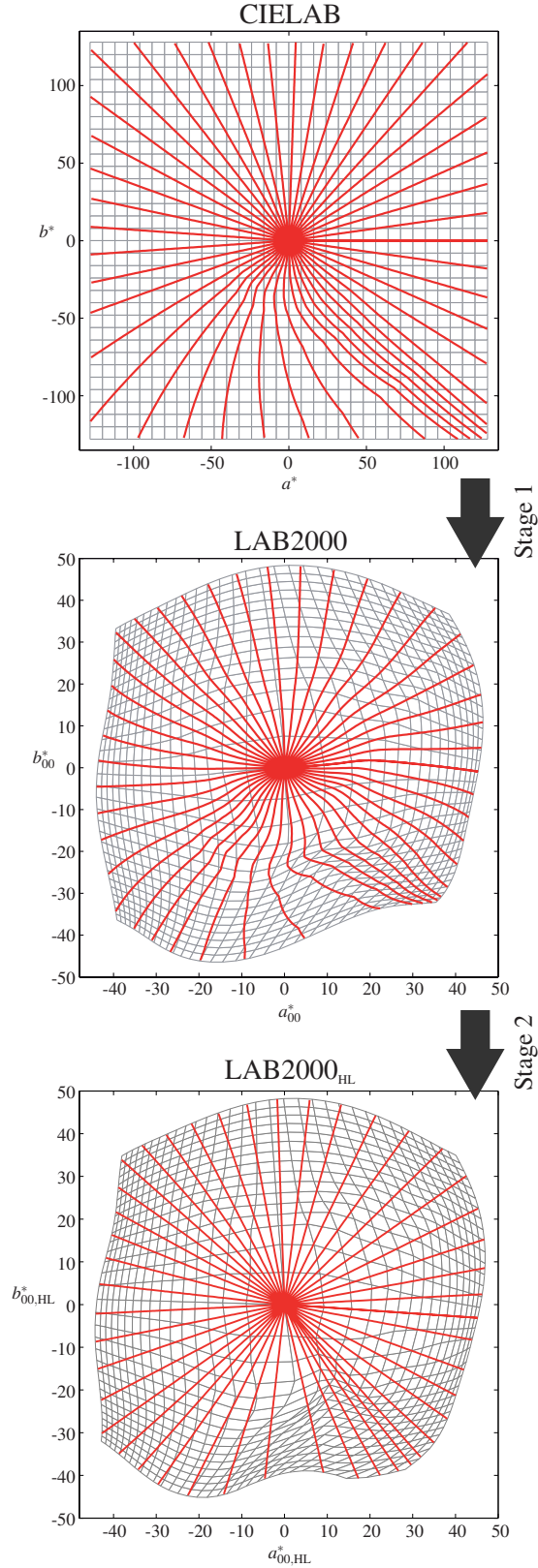


Figure 1. Color space transformation with subsampled lookup table grids (gray) at a constant lightness level. Subsets of the Hung and Berns curves of constant hue are shown in red (the hue correction is limited to the blue region).

subject to

$$\phi_i \leq \phi_{i+1}, \quad i = 0, \dots, 358, \quad (5)$$

$$\phi_{359} \leq \phi_0 + 360. \quad (6)$$

The constraints in Eqs. (5) and (6) ensure that the grid topology is unaffected by the optimization.

For each vertex of the CIELAB grid, the CIEDE2000 color differences to its four neighboring vertices are calculated. This is done only once before the optimization. During the optimization, the vertices of the LAB2000 grid are hue-corrected with varying sets of hue angles $\phi = \{\phi_0, \dots, \phi_{359}\}$. For each vertex of the resulting hue-corrected LAB2000 grids, the Euclidean distances to its four neighboring vertices are computed.

The *color pair disagreement* δ between two vertices is defined as

$$\delta(\Delta E_{00}, \Delta E_{\text{Euc}}^\phi) = \left[\frac{\max\{\Delta E_{00}, \Delta E_{\text{Euc}}^\phi\}}{\min\{\Delta E_{00}, \Delta E_{\text{Euc}}^\phi\}} - 1 \right] \cdot 100\%, \quad (7)$$

where ΔE_{00} is the pre-computed CIEDE2000 color difference between the vertices on the CIELAB grid, and $\Delta E_{\text{Euc}}^\phi = \|\cdot\|_2$ is the Euclidean distance of the corresponding vertices on the hue-corrected LAB2000 (LAB2000_{HL}) grid.

Finally, the disagreement $d(\phi)$ we are minimizing is the mean of all $N \cdot 4$ color pair disagreements, where N is the number of grid vertices and each vertex has four neighbors.

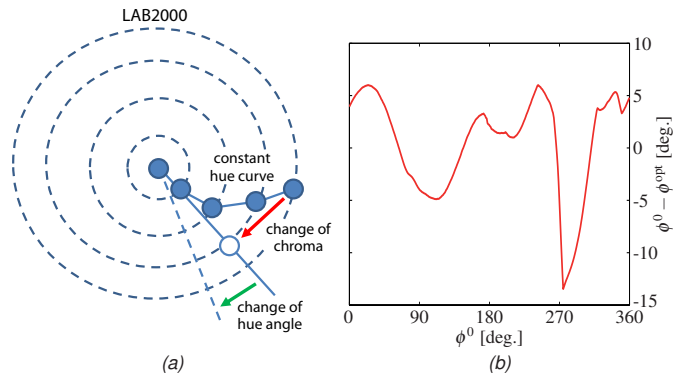


Figure 2. (a) Degrees of freedom for the hue correction of the LAB2000 color space: change of chroma, change of hue angle. (b) Difference between (equidistant) starting hue angles and optimized hue angles.

We used MATLAB to solve the optimization problem in Eq. (4) with starting angles $\phi^0 = \{0, 1, 2, \dots, 359\}$. Having found a set of optimized angles $\phi^{\text{opt}} = \{\phi_0^{\text{opt}}, \dots, \phi_{359}^{\text{opt}}\}$, we transformed the LAB2000 grid used to construct the lookup table for \mathcal{T}_{00} into the hue linear space as described above. The resulting grid in the LAB2000_{HL} space is shown in Fig. 1. The color space transformation $\mathcal{T}_{00,\text{HL}}$ from LAB2000 to LAB2000_{HL} is defined by mapping the vertices of the corresponding grids to each other. The result is a color lookup table; intermediate points are transformed using bilinear interpolation. A direct transformation from CIELAB to LAB2000_{HL} can be defined accordingly by mapping the initial CIELAB grid to the corresponding LAB2000_{HL} grid. Again, this can be expressed as a function composition: $\mathcal{T} = \mathcal{T}_{00,\text{HL}} \circ \mathcal{T}_{00}$.

Results and Discussion

In the following section we evaluate how the hue correction step affects the disagreement between CIEDE2000 differences and Euclidean distances. The effect of the hue angle optimization (Eqs. (4)–(6)) is of particular interest. Is the optimization approach superior to using equidistant iso-hue lines? How perceptually uniform is the new space compared to the IPT color space?

To quantify the disagreement in LAB2000_{HL}, we generated two million random color pairs covering the CIELAB space with $L^* \in [0, 100]$, $a^*, b^* \in [-105, 105]$. The colors of each pair had a maximum distance of 5 CIELAB units as required for the CIEDE2000 formula [19]. Using \mathcal{S} , we transformed the color pairs to LAB2000_{HL} and calculated their Euclidean distances. The result is a set of two million CIEDE2000 distances and the corresponding Euclidean distances in LAB2000_{HL}. We evaluated the disagreement using Eq. (7) and the standardized residual sum of squares (STRESS) index [21]. The STRESS index allows a statistical evaluation of color-difference formulas based on the F-test.

Table 1 shows the mean and maximum disagreements and the STRESS values for the evaluated color spaces. The IPT color space shows very large disagreement with CIEDE2000 differences if the above CIELAB test set is used. Therefore, two million color pairs from the sRGB gamut were used to evaluate this color space.

Color Space	Mean	Max.	STRESS
LAB2000	2.3%	117.5%	3.06
LAB2000 _{HL} (not optimized)	7.6%	371.2%	11.59
LAB2000 _{HL} (optimized)	5.0%	221.0%	9.43
IPT (sRGB gamut)	–	–	28.46

Table 1. Mean and maximum disagreements according to Eq. (7) and STRESS values for two million random color pairs.

Figure 4 (a) shows iso-distance contours with $\Delta E_{00} = 1$ projected onto the CIELAB a^*b^* -plane. Contours located close to the lightness axis are very small. Their size increases with increasing chroma. They are far from being equally sized circles, which would indicate a perceptually uniform color space. The elliptical contours are rotated counterclockwise in the blue CIELAB region due to the rotation term of the CIEDE2000 formula.

Figure 4 (b) shows the corresponding iso-distance contours in the LAB2000 space. As shown in Table 1, the disagreement between Euclidean distances in LAB2000 and CIEDE2000 differences is extremely small. This is confirmed by the CIEDE2000 iso-distance contours in Fig. 4 (b), which are almost circles and of similar size throughout the a^*b^* -plane, indicating a nearly perceptually uniform color space (assuming that CIEDE2000 estimations accurately reflect perceived color differences). Only in the blue region the iso-distance contours are elliptical. Here, the Gaussian curvature of CIELAB with the CIEDE2000 formula is significantly different from zero, and an embedding into a Euclidean space is impossible [10].

Although the hue correction increases the mean disagreement, it is still rather small (7.6% without and 5.0% with optimization), possibly because the hue correction is limited to the blue region of the color space. However, the maximum disagreement increases significantly from 117.5% to 371.2% without optimization. The optimization step reduces the maximum disagreement to 221.0%. The optimization also decreases the STRESS index from 11.59 to 9.43.

This improvement is statistically significant on a 95% confidence level according to the F-test.

The largest angle shift caused by the optimization occurs in the blue region of the color space as seen in Fig. 2 (b). Hue-chroma plots shown in Figs. 3 (a)–(d) illustrate how the multigrid optimization and the subsequent hue correction distort the color space. Subsets of the constant hue curves used for the hue correction are shown in red. The grids shown in gray were used to construct the transformation \mathcal{S} . It is evident that the hue angle optimization expands the color space around a hue angle of 270° (Fig. 3 (d)). In this region, where the corresponding CIEDE2000 iso-distance contours differ significantly from equally sized circles (see Fig. 4 (c)), the disagreement is largest.

Finally, we compare the perceptual uniformity of our space with the IPT color space [5], which is also designed to be hue linear based on the Hung and Berns data. Note that the IPT hue correction is not limited to the blue color region. CIEDE2000 iso-distance contours in the $L^* = 60$ plane were transformed to IPT and are shown in Fig. 4 (d) as projections onto the plane of constant I . The contours are elliptically shaped and differ significantly in size depending on their location in the color space, which indicates a lack of perceptual uniformity. This is confirmed by the corresponding STRESS value in Table 1 (28.46), which is significantly higher than those for the LAB2000_{HL} spaces.

This is not surprising, because the IPT space was designed for simplicity: The transformation from CIEXYZ (CIEDE65/2°) into IPT consists of two 3×3 matrices and a power function to model non-linearities — there are too few degrees of freedom to incorporate the visual data completely. In contrast, a lookup table approach does not rely on predefined functions, and it can be accurately adjusted to visual data due to its many parameters (the nodes of the grids).

Conclusions

We proposed a two-stage transformation of a color space into a hue linear color space with a high degree of perceptual uniformity. The transformation is expressed as a color lookup table to enable convenient integration in gamut mapping algorithms.

Our experiments with CIELAB and the CIEDE2000 formula show that the mean disagreement between CIEDE2000 differences and Euclidean distances in the new space increases only moderately compared to a previous approach without hue correction. However, the maximum disagreement is significantly larger with hue correction than without hue correction.

The perceptual uniformity and the hue linearity of the resulting space depend on the underlying color-difference formula and the constant hue data that are used. The CIEDE2000 formula employed in our calculations shows significant disagreement with visual data, and the constant hue data were interpolated and extrapolated from experimental data. In addition, our hue correction is limited to the blue region of the color space. We therefore believe that the space can be further improved using more accurate hue data and a color-difference formula that reflects visual data more closely. Despite these shortcomings, the color space in its current form can be used in applications that require hue linearity and a high degree of perceptual uniformity.

It would be interesting to verify the performance of the color space using unknown visual data, i.e., psychophysical data that were not used to fit the underlying color-difference formula.

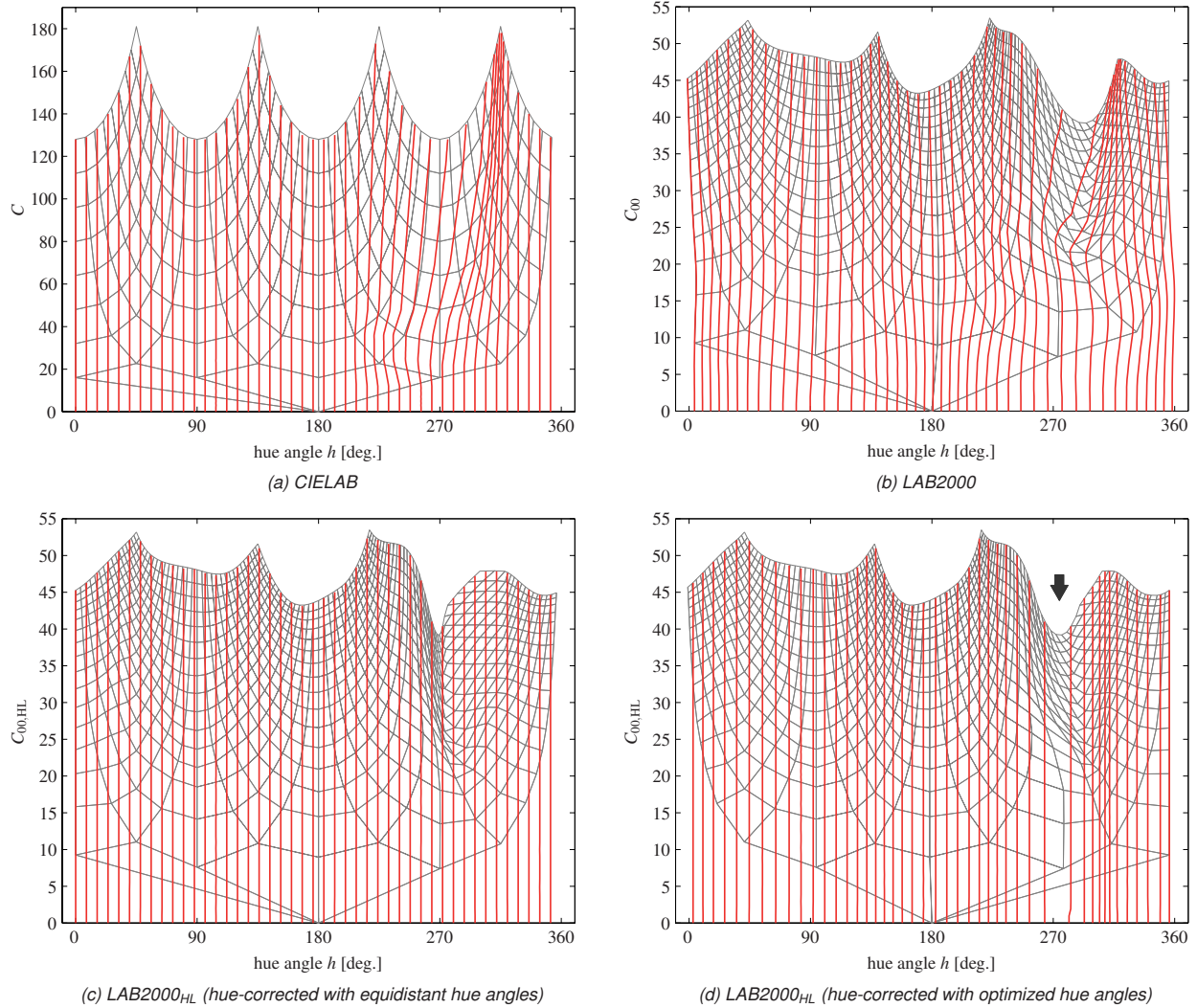


Figure 3. Hue-chroma plots showing subsets of the Hung and Berns curves of constant hue (red) in (a) the CIELAB color space (b) the approximately perceptually uniform LAB2000 color space (c) the hue-corrected LAB2000_{HL} color space (without optimization) (d) the hue-corrected LAB2000_{HL} color space (with optimization). Subsampled versions of the corresponding lookup table grids are shown in gray.

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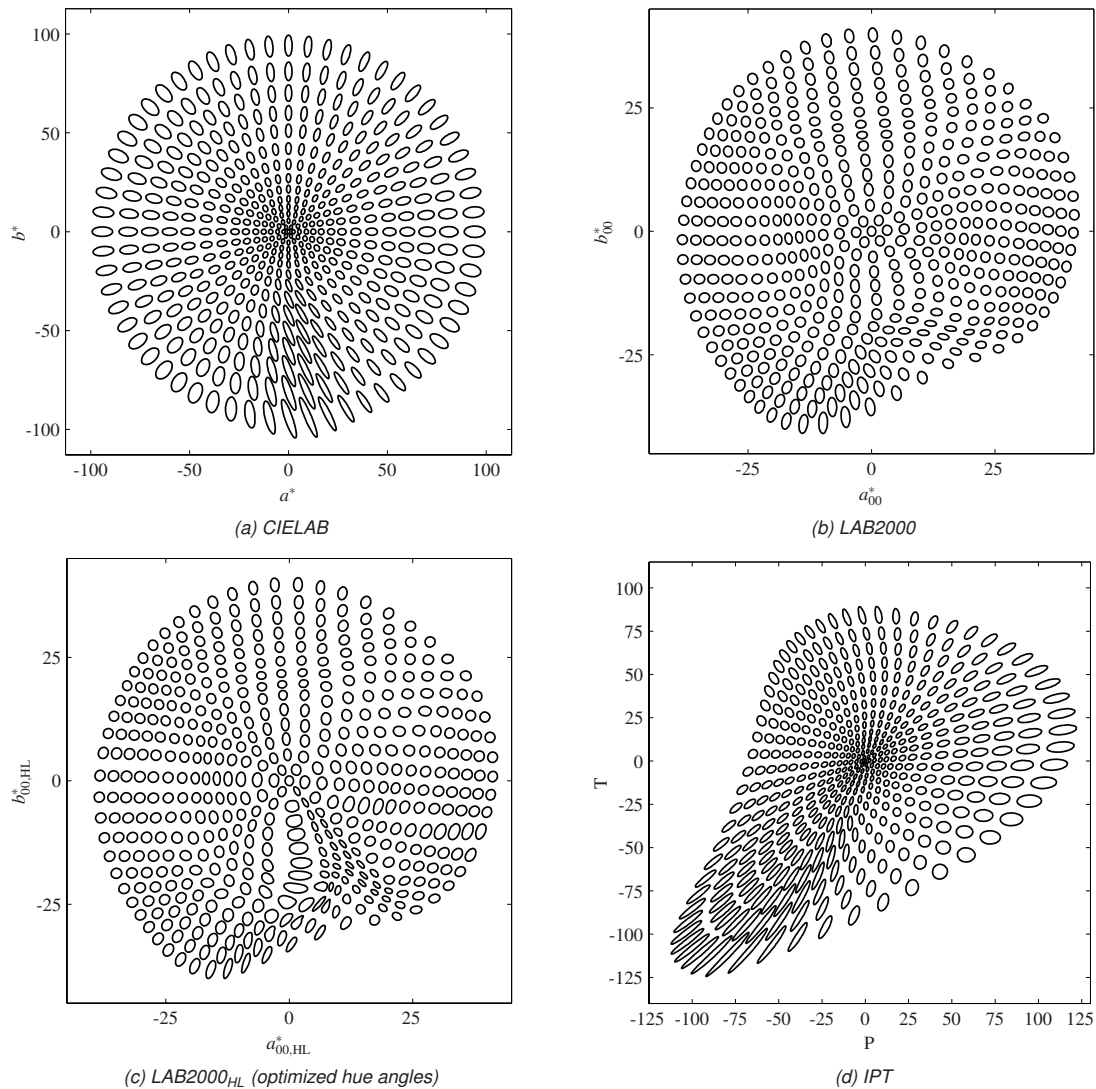


Figure 4. CIEDE2000 iso-distance contours in (a) CIELAB (b) the approximately perceptually uniform LAB2000 color space (c) the hue-corrected LAB2000_{HL} color space (with optimization) (d) IPT. Each point on a contour has a computed distance of $\Delta E_{00} = 1$ to the center of the contour.

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Author Biographies

Ingmar Lissner received his engineering degree in Computer Science and Engineering from the Hamburg University of Technology (Germany) in 2009. He currently is a research assistant and doctoral candidate at the Institute of Printing Science and Technology of the Technische Universität Darmstadt (Germany). His research interests include color perception, color reproduction, and high dynamic range imaging.

Philipp Urban has been head of an Emmy-Noether research group at the Technische Universität Darmstadt (Germany) since 2009. His research focuses on spectral-based acquisition, processing, and reproduction of color images considering the limited metameric and spectral gamut and low dynamic range of output devices. From 2006–2008 he was a visiting scientist at the RIT Munsell Color Science Laboratory. He holds a MS in mathematics from the University of Hamburg and a PhD from the Hamburg University of Technology (Germany).