Spectral Gamut Mapping Framework
based on Human Color Vision

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Abstract
A new spectral gamut mapping framework is presented. It adjusts the reproduction, choosing spectra within the printer’s gamut that satisfy colorimetric criteria across a hierarchical set of illuminants. For the most important illuminant a traditional gamut mapping is performed and for each additional considered illuminant colors are mapped into device and pixel dependent metamer mismatch gamuts. A computational separation method is proposed in order to test the framework. Utilizing this separation method on a seven channel printing system, experiments allowed a deeper view on the structure of the device and pixel dependent metamer mismatch gamuts and the possible directions in color space in which a potential metameric gamut mapping transformation could map out-of-metameric-gamut colors.

Introduction
In recent years spectral acquisition has become an active research field. Today’s technology is able to capture high resolution multichannel images with very small spectral estimation error. This technology is employed by museums for artwork reproduction and for archiving applications. Wide-gamut multicolour printers are used within traditional color management to create accurate colorimetric reproductions that match originals under a single illuminant. For some applications it can be desirable for reproductions to match originals under multiple illuminants. In such cases a spectral reproduction is needed. A basic limitation of such a reproduction is the physical ability of printing devices to reproduce spectrally. The spectral printer gamut is much smaller than the space of all natural reflectances. A lower bound of the dimensionality of natural reflectances can be determined through analysis of multiple spectral databases [1]. Only by looking onto the dimensionality difference does it become obvious that the majority of spectral reflectances cannot be reproduced without spectral error on a typical printer. It becomes necessary to map the unprintable spectra into the spectral gamut of the printer. Such a mapping is not unique and an optimal transformation strongly depends on the specific application. In recent years various metrics in spectral space have been proposed [2, 3]. To show an advantage compared to traditional color management, spectral reproduction should be for one illuminant as visually correct as a colorimetric reproduction and for other illuminants superior. An approach has been proposed, combining a mapping in a perceptual color space based on one illuminant and a spectral mapping within the corresponding three dimensional device metameric black space [4, 5, 6, 7].

In this paper we are presenting a spectral gamut mapping framework that adjusts a reproduction so that it matches the original under multiple illuminants considering properties of human color vision.

The Spectral Gamut Mapping Framework Terminology
In order to explain the spectral gamut mapping framework we use the common terminology of discrete spectra, resulting from a sampling of the continuous spectra at N equidistant positions within the visible wavelength range from 380 nm to 730 nm. Each reflectance spectrum is a N-dimensional vector \( r \in [0,1]^N \) and the set of illuminants for which the reproduction has to be adjusted is a set of N-dimensional vectors representing the spectral power distributions of the illuminants \( I_1, \ldots, I_N \in \mathbb{R}^N \).

In the following text we use the observer’s CIEXYZ tristimulus \( X(r,I) \) as a function of the reflectance \( r = (r_1, \ldots, r_N) \) and the illuminating illuminant \( I = (I_1, \ldots, I_N) \):

\[
X(r,I) = \frac{1}{\sum_{n=1}^{N} \gamma_n} \left( \sum_{n=1}^{N} \xi_n r_n, \sum_{n=1}^{N} \eta_n r_n, \sum_{n=1}^{N} \zeta_n r_n \right)^T
\]

(1)

where \( \xi, \eta, \zeta \) are the CIE color matching functions for the 2° or 10° observer, respectively. The color space transformation from CIEXYZ into the nearly perceptually uniform CIELAB color space is denoted by \( L : \text{CIEXYZ} \rightarrow \text{CIELAB} \) and the inverse function by \( L^{-1} : \text{CIELAB} \rightarrow \text{CIEXYZ} \).

The set of all reflectances that result in the CIELAB value \( x \) for an illuminant \( I \) is called metameric reflectance set and will be denoted by

\[
M(x,I) = \{ r \in [0,1]^N | L(X(r,I)) = x \}
\]

(2)

The spectral printer gamut, which is the space of all printable spectral reflectances of the given device, is denoted by \( G \subset [0,1]^N \).

Methodology

Step 1: In a first step we calculate for each considered illuminant a CIELAB image from the given multispectral image. If \( S \) is the set of all reflectances within the multispectral image we obtain \( L(X(S,I^1)), \ldots, L(X(S,I^n)) \).

Step 2: The main idea is to select an application-dependent illuminant for that the spectral reproduction shall be as good as a colorimetric reproduction (e.g., CIE-D50 if we want to be consistent with the ICC [8]). We denote this illuminant the base illuminant and it should be the first illuminant \( I^1 \) in our list of considered illuminants. For this illuminant a traditional gamut mapping transformation \( \Gamma_{\text{Gamut}} \) [9] has to be performed that transforms each pixel color of the corresponding CIELAB image into an in-gamut CIELAB color

\[
\Gamma_{\text{Gamut}}(\mathcal{G}) : \text{CIELAB} \rightarrow \mathcal{G}
\]

(3)
where \( L = L(X(G,1)) \) is the printer's CIELAB gamut, which is used as a parameter variable of the gamut mapping algorithm. As a result of this transformation all pixels of the new CIELAB image are within \( L(X(G,1)) \), i.e.

\[
\Gamma_{\text{map}} \left[ L(X(G,1)) \right] \left( L(X(S,1)) \right) \subseteq L(X(G,1))
\]  

(4)

It should be noticed that \( \Gamma_{\text{map}} \) is not limited to pixel-wise gamut mapping methods, also spatial gamut mapping methods can be used, see [10] for a comparative overview.

**Step 3:** In this step the reproduction is adjusted for the second illuminant. A traditional gamut mapping cannot be used again because it cannot be ensured that for an image pixel the gamut-mapped CIELAB color under the second illuminant combined with the gamut-mapped CIELAB color for the base illuminant can be reproduced by in-gamut spectra. As a consequence we have to deal with pixel dependent gamuts rather than a single global gamut such in the previous step.

Let \( r \) be a pixel reflectance of the multispectral image and \( x_{1}(r) = \Gamma_{\text{map}}[L(X(G,1))](L(X(r,1))) \) be the CIELAB color under the base illuminant resulting from the traditional gamut mapping. The CIELAB color corresponding to \( r \) for the second illuminant can only be mapped into the metamer mismatch gamut (see Figure 1) resulting from the intersection of the device's spectral gamut \( G \) and the metamer reflectance set \( M(x_{1}(r),1) \) (see eq. (2)).

We denote such a mapping into the device dependent metamer mismatch space by

\[
\Gamma_{\text{map}}[\mathcal{M}] : \text{CIELAB} \rightarrow \mathcal{M}
\]  

(5)

where \( \mathcal{M} \) is the device and pixel dependent metamer mismatch gamut, which is used as a parameter variable

\[
\mathcal{M} = L(X(M(x_{1}(r),1) \cap G,1))
\]  

(6)

\( \Gamma_{\text{map}} \) can utilize transformations that are related to human color vision like minimizing color difference or preserving the hue angle. Minimizing color differences is similar to minimizing the metameric index and is already described by Tseng and Bems [11]. Some possible \( \Gamma_{\text{map}} \) transformations can be:

1. \( \Gamma_{\text{map}}[\mathcal{M}](x) = \arg \left( \min_{y \in \mathcal{M}} \left( \Delta E_{\text{ab}}(x,y) \right) \right) \)

2. \( \Gamma_{\text{map}}[\mathcal{M}](x) = \arg \left( \min_{y \in \mathcal{M}} \left( \Delta E_{\text{ab}}(x,y) \right) \right) \)

3. \( \Gamma_{\text{map}}[\mathcal{M}](x) = \arg \left( \min_{y \in \mathcal{M}} \left( \Delta E_{\text{ab}}(x,y) \right) \right) \)

Transformations 2 and 3 utilize the \( k_{L}, k_{C} \) and \( k_{H} \) coefficients of the CIE94 [12] and CIEDE2000 [13] color distance formulas. By setting \( k_{L}, k_{C} > k_{H} \), distances in hue direction are weighted larger [14][15]. In case of setting \( k_{L}, k_{C} = 2 \) and \( k_{H} = 1 \) maintaining hue accuracy has twice the importance as lightness and chroma.

**Step 4_{n}(n+1):** For each additional illuminant \( \tilde{\ell} \) the pixel CIELAB color \( L(X(r,\tilde{\ell})) \) can only be mapped onto the device and pixel dependent metamer mismatch gamut, which results from the intersection of the spectral printer gamut \( G \) and the intersection of all metameric spectra of the corresponding gamut-mapped CIELAB colors \( x_{1}(r), \ldots, x_{n}(r) \) under the previous considered illuminants, i.e.

\[
\mathcal{M} = L(X(\bigcap_{j=1}^{n-1} M(x_{j}(r),1) \cap G,\tilde{\ell})).
\]  

(7)

The same transformation \( \Gamma_{\text{map}} \) can be used to map the pixel CIELAB color onto the metamer mismatch gamut as in Step 3.

For each illuminant only transformations in a three dimensional space have to be calculated. The results of these transformations are used as parameters of transformations for the next illuminant. In this way the reproduction is adjusted hierarchically to a set of given illuminants. For a pixel reflectance \( r \) the spectral gamut mapping can be summarized as follows:

\[
x_{1}(r) = \Gamma_{\text{map}} \left[ L(X(G,1)) \right] \left( L(X(r,1)) \right)
\]

\[
x_{2}(r) = \Gamma_{\text{map}} \left[ L(X(M(x_{1}(r),1) \cap G,1)) \right] \left( L(X(r,1)) \right)
\]

\[
\vdots
\]

\[
x_{n}(r) = \Gamma_{\text{map}} \left[ L(X(\bigcap_{j=1}^{n-1} M(x_{j}(r),1) \cap G,\tilde{\ell}))) \right] \left( L(X(r,\tilde{\ell})) \right)
\]

If enough linearly independent illuminants are considered, so that the matrix

\[
\Omega = \left[ \begin{array}{c} \Omega_{1} \\ \vdots \\ \Omega_{n} \end{array} \right], \quad \text{where} \quad \Omega_{i} = \frac{1}{\ell_{i}} \left[ \begin{array}{c} x_{i}(r_{1}) \\ \vdots \\ x_{i}(r_{n}) \end{array} \right]
\]

(8)
has rank $N$, than an error-free reconstruction of in-spectral-gamut reflectances can be performed using the pseudoinverse

$$r_{\text{recon}} = (\Omega^T \Omega)^{-1} \Omega^T \begin{bmatrix} L^{-1}(x_1(r)) \\ \vdots \\ L^{-1}(x_n(r)) \end{bmatrix}. \quad (9)$$

If additionally $\Gamma_{\text{det}}$ and $\Gamma_{\text{tran}}$ leave in-gamut colors unchanged within each CIELAB color space for the considered illuminants the proposed framework leaves in-gamut spectra unchanged as well. See Figure 2 for a flowchart of the framework.

It should be noticed that the resulting in-gamut reflectances are not only depending on the considered illuminants but also on their order. Another property of the proposed spectral gamut mapping can be derived directly from eq. (1): If the reproduction matches the original under a set of illuminants than it matches the original under each mixture of these illuminants [18]. This property can be very useful if the viewing conditions are blending continuously between a fixed set of illuminants.

![Flowchart of the spectral gamut mapping framework.](image)

**Figure 2.** Flowchart of the spectral gamut mapping framework. The multispectral image is transformed into a CIELAB image and for a set of application dependent illuminants the first CIELAB image for the base illuminant (illuminant 1) is transformed into the metamer gamut by a traditional gamut mapping. The remaining CIELAB images for the other illuminants are transformed pixel-wise onto the device and pixel dependent metamer mismatch gamuts resulting from the previous gamut mapped images. The transformations are related to human vision, e.g. by minimizing the CIEDE2000 distance. From the resulting ingamut CIELAB images an in-spectral-gamut multispectral image can be reconstructed, if sufficient linearly independent illuminants are used.

**Computational Separation for Testing the Gamut Mapping Framework**

In general the separation process can be described as a concatenation of a spectral gamut mapping, and a printer model inversion. For a printer whose spectral response is characterized by a cellular Yule-Nielsen Spectral Neugebauer (CYSN) model a fast inversion method is described by Urban et al. [16]. The difficulty in realizing the proposed gamut mapping framework is the calculation of the pixel and device dependent metamer mismatch gamuts

$$L(X \left( \bigcap_{j=1}^{i-1} M(x_j(r), G_t) \cap G_t \right), \quad i = 2, \ldots, n. \quad (10)$$

The calculation of these gamuts for multiple illuminants and high resolution images seems impossible in reasonable time.

Therefore, we chose a different strategy that is completely computational and combines spectral gamut mapping as well as model inversion for the whole image in a single step. We assume that the spectral printer gamut can be reasonably described by the connection of a set of spectral gamuts that are defined by all CYSN sub-models containing 4 colors including black. This assumption has been already used by Tseng and Berns [11] for modeling a 6 color printer. For a CMYKRGB printer 20 sub-models have to be considered. Restricting the maximum number of overprints to 4 has an additional advantage since more overprints tend to behave unstable in terms of color accuracy. The separation method uses the color just noticeable distance (JND) of the human visual system (HVS) as well as the high quantization of typical printing devices [17, 18]. Using the traditional gamut mapping $\Gamma_{\text{det}}$ within a hue linearized [19] CIELAB color space for the base illuminant the CIELAB image is transformed into the metameric printer gamut. A 3D histogram is created for this image and for each sub-model the colorant space is sampled in 1% steps resulting in approximately 100 million different colorant combinations. For the 20 sub-models a total of 2 billion colors were transformed by the forward model for the base illuminant and tested using the 3D histogram for matching pixel CIELAB values of the already gamut-mapped image. For each colorant combination that matches a CIELAB pixel value for the base illuminant, the corresponding CIELAB value for the second illuminant is calculated using the forward printer model and compared with the corresponding pixel CIELAB value for the second illuminant using the function which the metameric gamut mapping transformation $\Gamma_{\text{tran}}$ is based (e.g. CIEDE2000 or CIE94 with special weight on the hue-difference). This is also done for all other illuminants and the colorant combination was chosen for the separation, which minimizes a weighted sum of these function values. The weights can be chosen according to the importance of the illuminant within the considered illuminant set. A flowchart of the computational separation method is shown in Figure 3.

The whole separation process needs approximately 5 min. for a 22-megapixel image (printing in the style of Vincent van Gogh’s Church at Auvers [22]) on an Intel Q6600 quad-core processor using a performance optimized C++ implementation. It has to be noticed that the computational time depends on the image content and the distribution of the 3D histogram as well as on the number of metamer pixel colors for the base illuminant. Even by using a 24-megapixel image that consists of completely random colors spanning the whole color space for the base illuminant the computational time did not exceed 10 min. on the described hardware.

A drawback of the proposed separation method is the disregard of any spatial properties of the separation. Neighboring pixels with nearly equal spectra can result in complete different separations. As a consequence banding artifacts in the print can occur especially for noise-free source images. For noisy images captured by a multispectral camera such artifacts are not observed. Nevertheless, in future work spatially smoothing constraints within the printer’s control value space need to be added to the separation method.
Figure 4. Examples of images separated for printing with high color inconstancy. After printing, visual inspections confirmed that color changes of the original across illuminants were mimicked by the reproduction.

Figure 3. Flowchart of the computational separation technique that is used to test the spectral gamut mapping framework.

Experimental Setup
An HP Z3100 Photo printer was used and controlled by a Onyx Production House RIP (Version 7). The metameric gamut under illuminant CIE-D50 of the printer can be seen in Figure 1. Only the CMYRGB ink subset of the 12 available inks were used, since the other inks, mainly different black types and a gloss enhancer, do not contribute significantly to the spectral variability. The matix used was Felix Schoeller (H74261) 270g/m² paper that does not include optical brightener. Each of the sub-model has $4^2 = 256$ cells with optimized positions of the cell primaries, according to the method of Chen et al. [20]. To characterize the printer a total of 7725 patches were printed and measured.

Results
To test our framework we used a hue and lightness preserving chroma clipping as the traditional gamut mapping method $\Gamma_{\text{clipped}}$. For the metameric gamut mapping $\Gamma_{\text{metamer}}$ a simple minimizing $\Delta E_0$ was employed. We printed various images, e.g., the highly color inconstant METACOW [21]. Additionally, we reproduced paintings that include pigments with challenging spectral reflectances such as cobalt blue and ultramarine blue (see Figure 4). The color changes of the original across the considered illuminants (CIE-D65 and CIE-A) were mimicked by the reproductions. A detailed analysis how the printing system with the proposed spectral separation framework is embedded into an end-to-end spectral reproduction system is made in a further CGIV 2008 paper [22]. In this paper quantitative results are given in terms of CIEDE2000 color differences for all considered illuminants.

In the present paper we were more interested in the structure of the device and pixel dependent metamer mismatch gamuts and the possible directions in color space in which a potential metameric gamut mapping transformation $\Gamma_{\text{metamer}}$ could map out-of-metamer gamut colors.

For this reason we calculated a separation of the METACOW image for the described printing system with base illuminant CIE-D65 and second illuminant CIE-A. The METACOW image was constructed in a way that the left side of each row has spectral reflectance properties measured from a GretagMacbeth ColorChecker and the right side of each row is a metameric match under CIE-D65 that maximizes color differences under illuminant CIE-A. All CIELAB colors of the image were within the CIELAB device gamut for illuminant CIE-D65 (except for the highlights and the black areas, which have lightness values greater than paper white or smaller than the black ink, respectively). Therefore, our traditional gamut mapping method $\Gamma_{\text{clipped}}$ basically did not change any chromatic colors. We picked two pixels from each row, which lie opposite sides and are metameric under illuminant CIE-D65. The corresponding device and pixel dependent metamer mismatch gamuts were plotted in Figure 5 for illuminant CIE-A together with the corresponding pixel CIELAB colors for both pixels. It can be seen that most of the CIELAB colors under illuminant CIE-A from points located at the left side of each row can be printed by our system since these colors are located mostly within the device and pixel dependent metamer mismatch gamuts. Points located on the right side of the rows are mostly far outside of the device and pixel dependent metamer mismatch gamuts.

The position of these points relative to the metamer mismatch gamuts does not allow a hue preserving mapping. This can be seen especially for cow 2, 3 and 5. In contrast to traditional gamut mapping methods that mostly try to preserve hue this cannot be guaranteed for a mapping onto device and...
pixel dependent metamer mismatch gamuts.

A further interesting observation is that some device and pixel dependent metamer mismatch gamuts have larger chroma values than the corresponding pixel colors (see e.g. cow 16). A mapping onto such metamer mismatch gamuts would result in a chroma gain, which is also unusual for traditional gamut mapping methods.

In future work we want to conduct psychophysical experiments in order to test different metameric gamut mapping transformations $\Delta L_m$.

**Conclusion**

A spectral gamut mapping framework was proposed that hierarchically adjusts the reproduction for a set of considered illuminants. This adjustment consists of a traditional gamut mapping for a base illuminant and mapping onto device and pixel dependent metamer mismatch gamuts for the other illuminants. In case of considering enough linearly independent illuminants the resulting set of trim stimuli can be used to reconstruct in-spectral-gamut reflectances. Experimental results show that a hue preserving mapping onto device and pixel dependent metamer mismatch gamuts cannot be guaranteed and as a consequence hue shifts of the print compared to the original cannot be avoided if they are compared under a different illuminant than the base illuminant.

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**References**


**Author Biography**

Philipp Urban received his M.S. degree in Mathematics from the University of Hamburg in 1999 and his Dr. degree in the field of color science from the Hamburg University of Technology in 2005. From 1999 until 2006 he was part of the research group "Vision Systems" at the Hamburg University of Technology and worked for Ratio Entwicklung GmbH (ICC-member) where he developed color managing systems. Since 2006 he is a visiting scientist at the Munsell Color Science Laboratory at the Rochester Institute of Technology. His research interests are color science and multispectral imaging.
Figure 5. METACOW: Device and pixel dependent metamerism match gamuts under illuminant CIE-A, calculated for pixel pairs that are metameric under illuminant CIE-D65. Each pixel pair belongs to a cow and contains one pixel on the left side of the cow and one pixel on the right side of the cow. The CIELAB colors of each cow pixel pair under illuminant CIE-A are marked by "" for the left pixel and by "" for the right pixel. The contour line in each diagram marks the CIELAB gamut of the printer under illuminant CIE-A.